Let $\rho(x)$ be the linear density of a rod (in grams per centimeter) at the point $x$ centimeters from the left end **SCORE: _____ / 2 PTS**

(eg. $\rho(12) = 10$ means that at the point $12$ cm from the left end, the rod's density is $10$ grams per centimeter of rod).

What is the meaning of the equation $\int_{50}^{60} \rho(x)\, dx = 80$ ?

**NOTES:** **Your answer must use all three numbers from the equation, along with correct units.**
**Your answer should NOT use "$x$", "$\rho$", "integral", "antiderivative", "rate of change", "change" or "derivative".**
**Your answer should sound like normal spoken English.**

The part of the rod between $50$ cm from the left end to $60$ cm from the left end

has a mass of $80$ grams

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**In complete sentences, using proper English and mathematical notation,** state the Fundamental Theorem of Calculus (both parts) and the Net Change Theorem.

SCORE: _____ / 5 PTS

FTC 1: If $f$ is continuous on an interval containing $a$, and $g(x) = \int_a^x f(t)\,dt$,

then $g'(x) = f(x)$ for all $x$ on that interval.

FTC 2: If $f$ is continuous on $[a,b]$, and $F' = f$ on $[a,b]$,

then $\int_a^b f(x)\,dx = F(b) - F(a)$.

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NCT: If $F'$ is continuous on $[a,b]$,

then $\int_a^b F'(x)\,dx = F(b) - F(a)$.

Evaluate the following integrals, or explain why they can't be evaluated. **SCORE: _____ / 12 PTS**

[a] $\displaystyle\int_2^4 \frac{5x^2}{16+4x^6}\,dx$

$$= \frac{5}{16}\int_2^4 \frac{x^2}{1+\frac{1}{4}x^6}\,dx$$

1 $u = \frac{1}{2}x^3 \Rightarrow \frac{du}{dx} = \frac{3}{2}x^2 \Rightarrow \frac{2}{3}du = x^2dx$

$x = 4 \Rightarrow u = 32$

$x = 2 \Rightarrow u = 4$

$$\frac{5}{16}\frac{2}{3}\int_4^{32} \frac{1}{1+u^2}\,du$$

1 ½

1 ½

$$= \frac{5}{24}\arctan u\Big|_4^{32}$$

$$= \frac{5}{24}(\arctan 32 - \arctan 4)$$ ½

[b] $\displaystyle\int \frac{(3z^2-2\sqrt{z})^2}{18z^5}\,dz$

$$= \int \frac{9z^4 - 12z^{\frac{5}{2}} + 4z}{18z^5}\,dz$$

$$= \int \left(\tfrac{1}{2}z^{-1} - \tfrac{2}{3}z^{-\frac{5}{2}} + \tfrac{2}{9}z^{-4}\right)dz$$ 1

$$= \tfrac{1}{2}\ln|z| + \tfrac{4}{9}z^{-\frac{3}{2}} - \tfrac{2}{27}z^{-3} + C$$

½ 1 1 ½

[c] $\displaystyle\int_{-3}^{3} \frac{2t^3}{t^4-16}\,dt$

Integrand is not continuous (vertical asymptote) @ $t = \pm 2$

1

so FTC2 does not apply

½

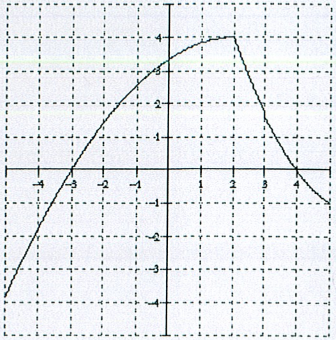
[d] $\displaystyle\int_{-1}^{1} \frac{\tan y}{1+\sin^2 y}\,dy$

$$\frac{\tan(-y)}{1+\sin^2(-y)} = \frac{-\tan y}{1+(-\sin y)^2} = -\frac{\tan y}{1+\sin^2 y}$$ 1

Integrand is odd and continuous, so integral $= 0$

½ ½

Let $p(x) = \int_{-2}^{x} f(t)\, dt$ , where $f$ is the function whose graph is shown on the right. **SCORE: _____ / 7 PTS**




[a] Write "I UNDERSTAND" to indicate that you understand that the graph shows $f$ , but that the questions below ask about $p$ .

[b] Find $p'(2)$ . **Explain your answer very briefly.**

$p'(2) = f(2) = 4$

1 1

[c] Find all critical numbers of $p$ . **Explain your answer very briefly.**

$p'(x) = f(x) = 0$ at $x = -3, 4$

1 1

[d] Find all intervals over which $p$ is both increasing and concave down at the same time. **Explain your answer very briefly.**

$p'(x) = f(x)$ is positive and decreasing on $[2, 4]$

2 1

(There are no critical numbers corresponding to $p'(x)$ DNE)

If $b(x) = \int_{x^4}^{4^x} \arctan(1+t^2)\,dt$ , find $b'(x)$ . **SCORE: _____ / 4 PTS**

$$b(x) = \int_{x^4}^{0} \arctan(1+t^2)\,dt + \int_{0}^{4^x} \arctan(1+t^2)\,dt = -\int_{0}^{x^4} \arctan(1+t^2)\,dt + \int_{0}^{4^x} \arctan(1+t^2)\,dt$$

$$b'(x) = -\frac{d}{dx}\int_{0}^{x^4} \arctan(1+t^2)\,dt + \frac{d}{dx}\int_{0}^{4^x} \arctan(1+t^2)\,dt$$

$$b'(x) = -\left(\frac{d}{d(x^4)}\int_{0}^{x^4} \arctan(1+t^2)\,dt\right)\frac{d(x^4)}{dx} + \left(\frac{d}{d(4^x)}\int_{0}^{4^x} \arctan(1+t^2)\,dt\right)\frac{d(4^x)}{dx}$$

$$b'(x) = -\left(\arctan(1+(x^4)^2)\right)\left(4x^3\right) + \left(\arctan(1+(4^x)^2)\right)\left(4^x \ln 4\right)$$

$$b'(x) = \underbrace{-}_{1}\overbrace{4x^3}^{1}\,\underbrace{\arctan(1+x^8)}_{\frac{1}{2}} + \overbrace{4^x(\ln 4)}^{1}\,\underbrace{\arctan(1+4^{2x})}_{\frac{1}{2}}$$